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Knotris: A New Game

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In 2019-2020, we began our knot mosaic research, sponsored by the Center for Undergraduate Research in Mathematics (CURM). To build the intuition to ask meaningful questions, we played with wooden knot mosaic tiles, created by Lew Ludwig for the UnKnot Conference, together with acrylic tiles that we created. We expected to make conjectures and prove theorems typical of pure mathematics research. Instead, while playing with tiles in our Seattle University dormitory, Chardin Hall one day, our play ignited a spark that inspired us to develop Knotris, a Tetris-like game that uses knot mosaic tiles instead of tetrominoes.

Let’s begin by introducing some knot mosaic vocabulary. A knot mosaic is a rectangular configuration created using the 11-knot mosaic tiles (Figure 1), and can be used to represent any knot (Figures 2 & 3). A knot mosaic is a rectangular configuration created using the 11 basic-knot mosaic tiles (Figure 1), and can be used to represent any knot (Figures 2 and 3).

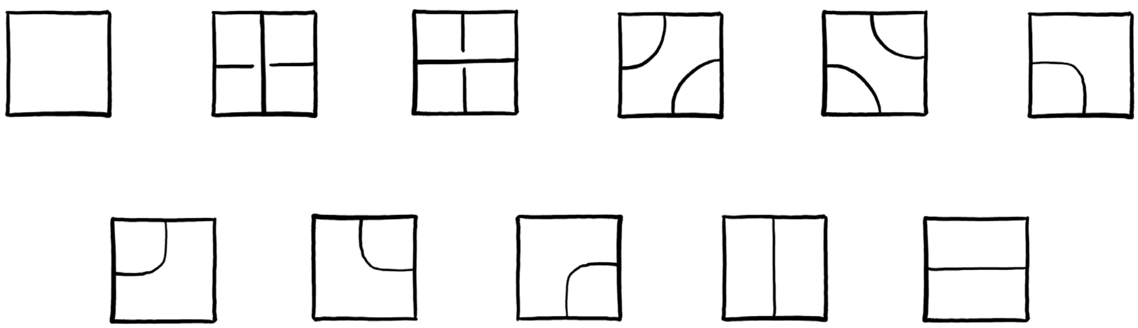

Figure 1 Eleven-knot mosaic tiles.

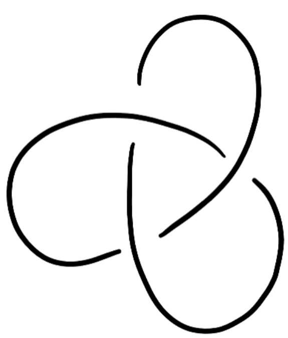

Figure 2 The knot diagram of a trefoil.

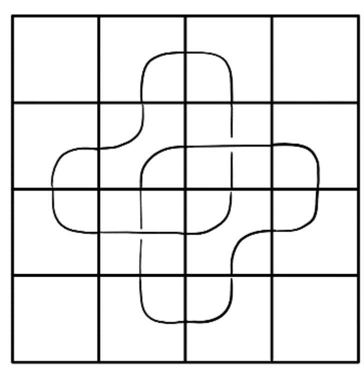

Figure 3 The knot diagram of a trefoil.

A connection point is present along any edge of a tile that a strand passes through (Figure 4). We can say a tile is “suitably connected” when each of its connection points touches a connection point of a contiguous tile.

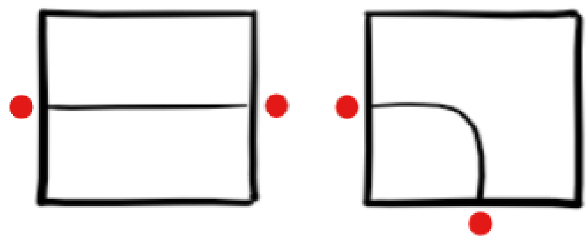

Figure 4 The connection points of a line and elbow tile indicated by red dots.

To inform our game design we drew from the classic video game, Tetris. In the rules of the game, a single tile falls at a time and can be rotated; the game has hold and drop tile features; the score increments when a row is cleared; and the game ends once a tile is placed beyond the heigh of the board. Beyond these parameters our game development was primarily guided by knot mosaic theory, probability analysis, and the intuition we built through play. In fact, intuition played an important role in determining what distribution of tiles would be supplied to the player. While playing with physical tiles, we found the seven-tile bag composed of one blank tile and two elbow, four-connected, and line tiles led to the most natural gameplay (Figure 5). Thus, we decided that tiles would be supplied to the player from a shuffled gamebag composed of three seven-tile bags, and upon emptying the gamebag would reset.

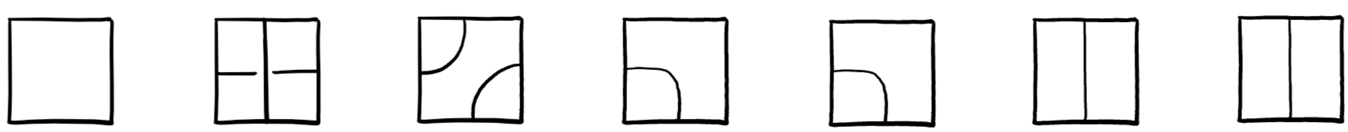

Figure 5 The seven-tile bag.

We can extend the definition of suitably connected to a 1x6 mosaic, which is a row in Knotris. Since we wanted more precise vocabulary to describe game states in our game, we expanded the definition of a suitably connected row to occur when two conditions are satisfied; the first is that a row is internally suitably connected, meaning that each of the connection points on an internal edge is satisfied (Figures 6 and 7). The second condition is that a row is externally suitably connected, such that the connection points along the upper edge of the row are satisfied by the row above it (Figure 8).

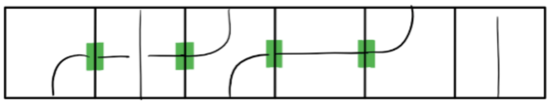

Figure 6 An internally suitably connected row.

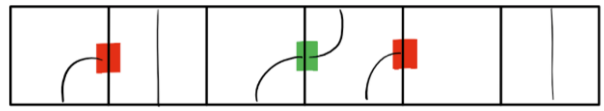

Figure 7 A not internally suitably connected row.

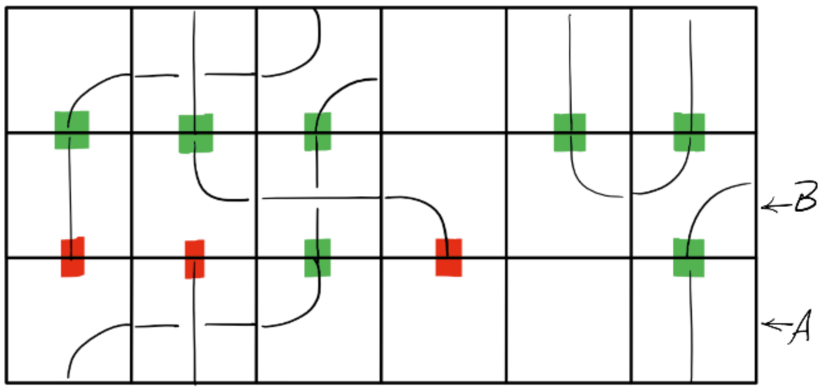

Figure 8 A knot mosaic where row A is not externally suitably connected, and row B is externally suitably connected.

To accrue points in Knotris the player must assemble suitably connected rows. If a row is placed such that it is internally suitably connected and not externally suitably connected, that row will remain on the board until a row is placed such that it is externally suitably connected. However, if a row is not internally suitably connected then that row and any rows below it that are not externally suitably connected to it will remain on the board for the duration of the game (Figure 9).

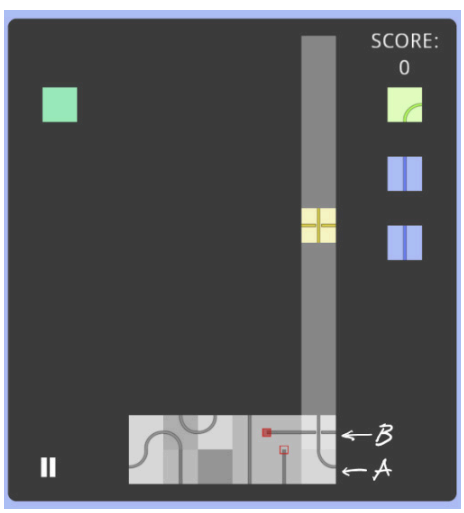

Figure 9 Row B is not internally suitably connected and row A is not externally suitably connected, so rows A & B are "stuck" on the board.

When a row clears, the score increments by 100 times the sum of the multiplier of each tile in the row. Initially, all tiles have a multiplier of one, but a player can increase the multiplier by creating knots and links or clearing multiple rows at once (Figure 10).

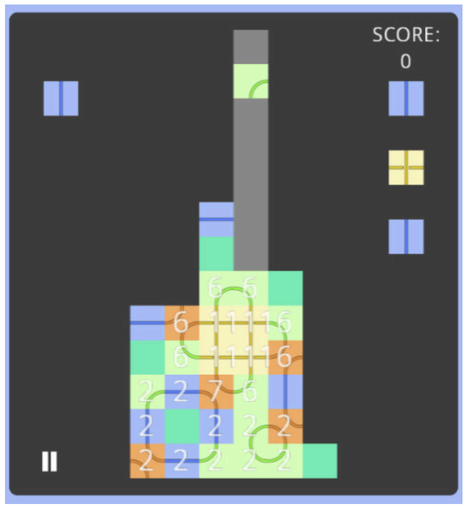

Figure 10 An example of increased tile multipliers by creating knots.

After we finalized the framework of the game, we wanted to know how challenging it would be to play, so we explored the tiles as combinatorial objects. We investigated how difficult it would be to fill holes on the board given varied assumptions about our tiles. This reveled how challenging it would be to clear rows in different game states, and this informed our strategies for gameplay. We spent weeks calculating the probability of filling one-, two-, and three-tile holes by hand drawing probability tree diagrams (Figure 11), which we later automated for more complex hole types and boundary conditions (Figures 12 and 13). We also evaluated the frequency of unique rows that could satisfy a given row boundary condition using our gamebag, and this provided additional insights into game strategies.

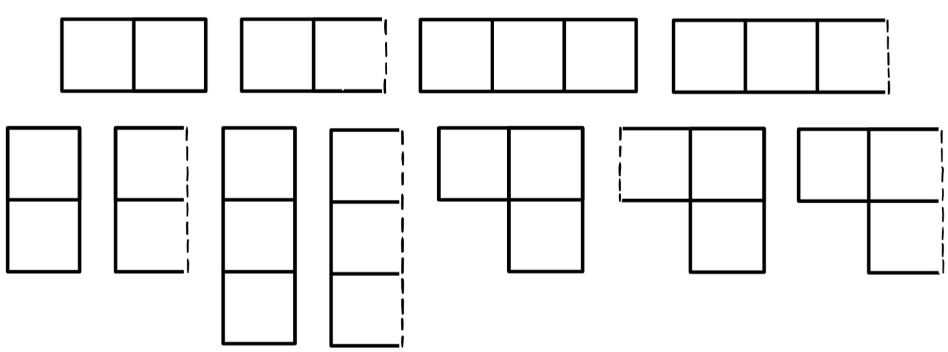

Figure 11 Examples of the two- and three-tile holes we analyzed. The dotted edges indicate there is no constraint on the edge.

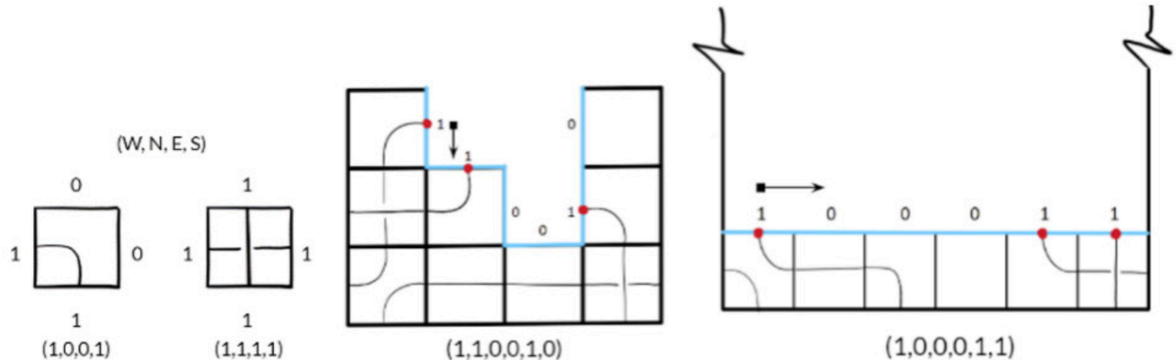

Figure 12 A boundary condition is the trace of the external edges of a tile, hole, or row. It is represented by n-tuples of 0s and 1s where 0 indicates there is no crossing point along the edge, and 1 indicates there is a crossing point along the edge.

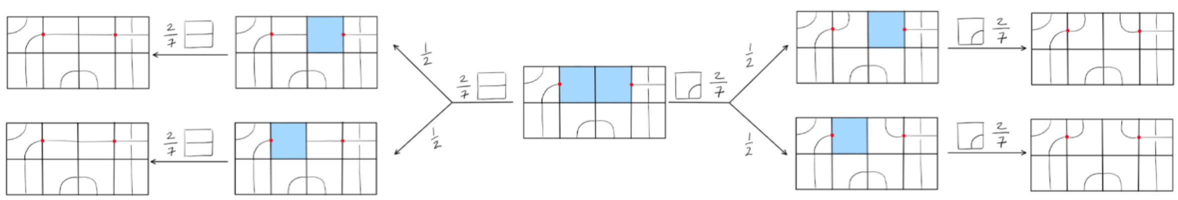

Figure 13 An example of the probability tree of a 1x2 hole with the boundary condition (1,0,0,1).

You can play Knotris at https://izook.github.io/knotris/. Be sure to fill out our feedback form: we would love to hear what you think! To learn more about the mechanics of the game and about knot mosaics, you can read our Math Horizons article, “Knotris: A Game Inspired by Knot Theory.” We would like to thank our collaborators at Bellevue College, Jen Townsend, Eden Chmielewski, Andre Dennis, and Kellen McKinney for a wonderful research experience; SUURJ for providing a platform to share our work; and CURM for funding our research.

Reference

Henrich A, Ionescu A, Mathews B, Ortega I, and Tesfaye K. 2021. KNOTRIS: A game inspired by knot theory. Math Horizons. 28(2): 8-11. DOI 10.1080/10724117.2020.1809238