July 2022

Box 16, Folder 13 - "Mathematics" [continued same as above]

Edwin Mortimer Standing

June 25th

Addition of Groups. –

Material for Memorising Addition of Groups

The Principle For the children we must search out the smallest difficulties & make them easier.
Teaching children is seeking out the minimum.

A Groups that do not Reach to 10

We could do it

1 + 1 ,	2 + 1 ,	3 + 1 ,	......	9 + 1 ..
1 + 2	2 + 2	3 + 2		9 + 2
1 + 3	2 + 3	3 + 3	. etc	9 + 3

we could give all these possible combinations but ~~we~~ on the contrary – we wish to draw out only those combinations which are necessary.

~~He~~ Eliminate the Ones

The child has had many exercises in counting one by one already – and counting what is it but adding one by one.

So we shall only take the groups of numbers – leaving out the ones – from two upwards.

We start at 4 – because 3 is make up of 2 + 1.
& 2 of 1+1. & we have eliminated Ones.
So the First analysis is 4 = 2+2.
[we could not form a four with any others having excluded the ones]

5 = 3 + 2 (for with 4 we should get a 1)
6 = 3 + 3 ; 4 + 2
7 = 5 + 2 . 4 + 3
8 = 6 + 2   5 + 3 . 4 + 4.
9 = 7 + 2   6 + 3   5 + 4

51

Within the limit of 10 there are no other combinations. This line represents the limits of 10.

So with regard to two numbers one comb^n
two others two "
two " three "

So Things fall into Order! So this table brings us to see intuitively that when we work with numbers in the proper way harmonious correspondences arise.

So ~~First~~ Next The combinations within 10 - in so far as they combine to form 10

10 = 8+2
10 = 7+3 The child has already made
10 = 6+4 these combinations with the
10 = 5+5. Rods in the Infants class.

Here not Quantity of Sums but Limitation & Order

The child has made many more additions than these. but now we are concerned with a certain principle - an order - a limitation. i.e. the only ones which can be made.

Not an Exercise on Counting but Composition & Decomposition of Numbers.

So we are here on a higher plane
We are getting above our Knowledge
Three Dimensional Knowledge

We shall let the child in a primitive period repeat & repeat - but in a higher stage we shall give only the minimum -

Second

~~Thus~~ Addition of Groups Astride of Ten

This is the line of separation - or repose. & 10 is always something restful or reposeful!

Every time we come to a 10 we feel as if we were masters of the Universe - the difficult part is that which surrounds the Ten

Thus - again omitting the ones -

9 + 2 = 11
9 + 3 = 12
9 + 4 = 13
9 + 5 = 14
9 + 6 = 15
9 + 7 = 16
9 + 8 = 17
9 + 9 = 18

So here we have 8 combinations and no more. If you come to 19 you have 10 + 9.

Now with eight

8 + 3 = 11
8 + 4 = 12
8 + 5 = 13
8 + 6 = 14
8 + 7 = 15
8 + 8 = 16

Have already had 8+9.
So here are 6 ~~7~~ combinations

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7 + 4 = 11
7 + 5 = 12
7 + 6 = 13
7 + 7 = 14

Four combinations

---

6 + 5 = 11
6 + 6. = 12

We must begin with 5 to get beyond 10
Two combinations

These are absolutely all. Whoever has done these can do any addition sum. There are no other combinations of numbers.. Thus 2. 4. 6. 8 Combin^s

Harmony from Analysis

2. 4. 6. 8. combinations.

So we see there is a harmony here, too, as there always is when we analyse in a just way

[cf. the Analysis in the Mass
The Crosses etc in Mystical Mass.]

Thus Numbers Become a Pleasure

Application of these Combinations in Snake Game

There is a pleasure doing the Snake Game now to show that no other combinations of numbers are possible.

Thus -

when we have added to our knowledge this same exercise has more interest & value.

So if you give Light & Interest - the child by these exercises will come to visualize & memorize the whole scheme of combinations

Snake Game serves to put in order these combinations

Summary of Exercises So Far

You see how many exercises have been born in this sphere of Arith^c. -

1) On the Groups of the Decimal System
2) On the Hierarchy
3) Counting - a) by units (infinite)
b) by groups (limited)

So now we can do any addition sum: when we have learnt to count 1) by ones
2) by groups.

"One Lesson Nature let me learn of Thee
Of Toil unsevered for Tranquility"

Tranquility of Mind. Arith^e

One important element of pleasure is a certain tranquility of mind - not to be anxious, pre-occupied about the future. - not to "gnaw the nail of hurry"

He is doing it because he likes to do it, & not because he is preparing to do addition & division & so on.

And therefore does these very things better when he does come to them.

Similarly with Coloured Designs ~~for~~ as a preparation for writing.

They do not think of them as a preparation for the future. These exercise give full satisfaction without ever thinking of the writing to come

So when the child is concentrated on these exercises I will not say they are preparing for these ~~exercises~~ operations but are preparing The Mathematical Mind.

Also Indirect Preparations

Addition

Example. You all come & bring me presents - cubes squares etc

Students come: she puts them all together.

There the addition is done.

Merely to organise the quantity according to the Principles of the D. System.

"See how all those people have left 2154 heads. I have only been counting up to 10 - yet up to thousands. - not in groups but by ones.

A Parallel Ex.

Thous	Hund	Tens	Units
''	'''''''	'''''	''''

2854

etc

"I can only count to 10. I can't do addition. I only know the hierarchy. I am in the lowest stage I know there are further more elevated things - But I can do it all the same - through this principle of the hierarchy -

Or again

23   1   3+9 = 10 + 2
49
64   2+4+9 : 15 = 10 + 5

39
21   9+8 = 17 = 10 + 7
38   3 10 5 + 4
4

In whatever way I abstract the tens I have added them

Some try different ways to abstract & sum up the tens

Toil inherent to Tranquility

Indirect Preparations

Arith c.

and

Writing

See. I

29/1

Parallel Exercises

When I speak of ‖ Ex. I mean those exercises which are separate one from another in themselves, but which however refer to one whole.

They are separate exercises & the children can begin with one or the other — as they do in the Casa de B. when they do the sensorial exercises. now choosing this material now that & all these exercises exist for a special teaching. Here it is the same thing

The Decimal System gives the fundamental & then many exercises which are different one fr. the other, & and taken all together they serve to illustrate the details & make a more profound preparation.

In this case they revolve about this detail of the re-grouping of the units — a thing simple but wh. needs maturing

As usual we Separate the Difficulties & give give them as interesting & fascinating wholes. — leading to joyous activity.

To understand this is but the first step

[Short Bead Stair — 1/29.

Colours attractive — but help.

at first child counts: after recognizes the quantity by colour fr. the before]

The Snake Game

Aim: First of all to practice counting – adding ~~little groups of bead-bars., but much etc.~~ the bead-bars; but instead of doing them singly he does them in groups.; & hence it is a real beginning in learning to add + know the results of two or more numbers added together 2 or more numbers, results which should be ^raised^ about 10.

Aesthetic This long many-coloured serpent.
Value of Snake Game
Comes a description of how to do it – substituting tens. This red orange golden serpent bit by bit eats up the coloured one.

These are very little sums, just over 10, sums already done in part with the rods on the C.d. B. & these are nothing more than counting with the rods & is an ex. on cd. actual do with the rods.

The interesting thing is to understand that there is nothing more complicated, & that is why it is easy to repeat & repeating one gets great agility in the D.S.

Seeing these things so evident there comes a profound persuasion not merely an acquaintance

Snake Game Contd.

Appears that the child is doing much more than it really is – for in the ordinary schools the mind carries along with it the weight of ~~all~~ that mental baggage which comes as the golden serpent grows. – 234 + 15 etc. – comes such a confusion that the mind is unable to sustain.

In the snake game the tens added up are put on one side for the time being.

Thus the difficulties are separated.

Control of Error – adding the pieces in groups making 10: at the end & comparing.

Thus we do two Exercises:

1) Adding these groups around 10.
2) Studying the composition of 10.

The Point of Reference

Thus the mind habituates itself to counting, always with ~~reference~~ a synthetic reference to 10 – as though everything is referred to one sole thing. & in this case (D.S) it is 10.

'Table of Pythagoras' for Addition

Geometric aspect of the Decimal System

pt. line. square cube.
pt line square cube
pt. line square cube.

8/29

82

How to carry the Mental Baggage

Other exercise in wh. the child learns to count - carrying the baggage with it as it goes along.

The difficulty of counting is always the same -
1) the Effort to count the single groups
2) At the same time carry all the accumulation of tens & groups of tens which are formed - grouping them according to their proper groups as we go along.

Counting the Thousand Chain

Needs patience - but the child has it and is something he has a need of - for you canto not force him to do it.

Linear Disposition of Numbers
Compared w. Geometric

This has certain advantages
The Geom. ~~vision~~ disposition gives a precise vision, idea of the quantity of units in a certain no - but the linear gives a different aspect.

Thus take 100 square & open it out & compare length with 10. It gives a clearer idea of the distance between 1 & 10, & 10 & 100.

So breaking cube to 10. squares & opening out into the Long Chain.

Exercise in Number

Stage I

1. Number Rods . w. cards
2. Spindles
3. Odds & Evens

Decimal System

1) Birds Eye view.
2) Making Numbers
3) Adding by the Dots.
4) Table of P. for addition 2
5) ~~Snake~~ Game 1
6) Geometric aspect of Decimal System.
   . 234, 2~~40~~, 240 .
5) Linear aspect of Numbers.
   cf. w. geometric
6). Making Nos with the cards.
   1 – 1000 & beyond .

Control of Error

Snake game.
Making the tens at the end

Working in the Details

The Embroidery on the Web

Addition of Groups

The addition table – cf Pythagoras for multiplⁿ

We may find ourselves in a position to add up groups of units. Suppose we wish to count the chairs in 3 different rooms. In one 4, another 7, the third 5: 4 + 7 + 5.

The adding can only be done in one way – by the Decimal System. What does this mean? It means that every time we arrive at 10 we have a different group. So we have to reduce the different groups into groups of ten.

$4 + 7 + 5 = 11 + 5 = 10 + 1 + 5 = 10 + 6.$

We have Two Difficulties

1) To know what results from the union of these groups of units

2) The other to realize that the group is always reduced to 10 + something.

Taking the First. The fact of knowing what results fr. the union of two groups is a fact of memory – something to be visualized

Here comes the Table of Additions – with all the possible combinations above ten – (between ten & twenty)

9 + 1	8 + 1	7 + 1	
9 + 2	8 + 2	7 + 2	"This line is
9 + 3	8 + 3		the limit of ten"
etc	etc		

Not so many as Table of Pythagoras: – but important

This is similar to what child has already done with Number-Rods but vaster.

These combinations must be placed in a clear manner. Then this is overcome. This difficulty remains the second — the grouping into tens.

The Snake Game

1) Arrange in a long variegated line.
2) Begin to add.
3) Each time we come to 10 or 10 + something put in a "yellow ten" and a black something
4) ~~3)~~ So in this way the yellow serpent goes devouring the coloured one — yellow with a black head!
4) Continue till all is "eaten up" — & one black (or black & white) head bar at the head
5) Group the tens — into hundreds if necessary & thousands.

So these variegated bits have all been brought into the Decimal System. The essence of this exercise is to find out "how many tens are in it". This is the really important thing.

The Proof of the Serpent Game. In order to fix this principle — that it is a searching for tens — we ~~m~~ can pick out all those bars which, placed together, make ten.

Sa 9 + 1 7 + 3 6 + 4 & so on. And thus we have the same result.

Then we group these tens we verify the hundreds

So when we have done these we can add up to any number (of the Mental Baggage comes separately)

Leading to the Explosion

Separate Exercises to Teach Details

Thus in Arith^c – as in writing – or in Reading we have separate exercises which teach the details – the exercises which lead up to the Explosion.

Third Dimensional Knowledge

The Study of Numbers.

An Intermediary Stage

Clear Presentation — a). The first Period – the very little ones.
Learning to count to 10
Number rods: spindles: discs
Exercises with the Rods

A Maturing Period — b). Then follows a Period in wh. child gives itself to a Study of Numbers & the Formation of numbers. – which is comparable to the study of "words. words words!" This stage is v. important & preparatory to :–

Utilization — © The Third Stage – when it utilizes its knowledge of numbers for operations

Contrast with Ordinary Method.

A Superficial knowledge of numbers does not suffice to start this study.
In ordinary methods they give or try to give a clear knowledge or understanding of numbers in order to utilize it immediately. But we put in an intermediate period between the clear knowledge & its utilization.
This is the most important period of all – a maturing period – When we have given the first knowledge (a) this formative knowledge (b) can only be done by the child itself. When he has learned something then he must exercise himself. When & How will be dictated by the Inner Force of His Mind

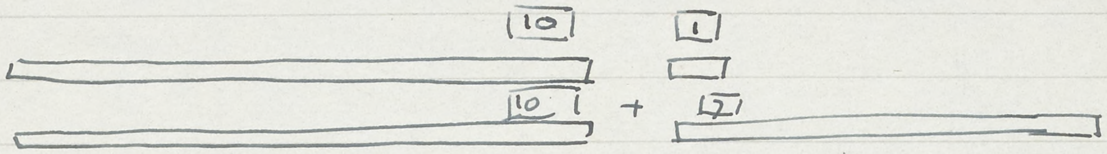
So We give a Material which

(a) Gives very clear ideas
(b) Lends itself to the manipulation of this idea

The Point of Departure

Thus to Carry on:—

Child has come to stage of Adding with Rods



and so on up till 19.

This is the Point of Departure

Important Principle.

The Success of the following period depends upon whether the previous period has been sufficiently done.

They may do many exercises with the 3 first steps (Rods, Spindles, Discs) or just abstractions as long as they like

After this comes a very different material.

Beads – Now a very different form of Material

Short Bead Stair

It relates to the Rods (Hook)

Differences

Colours – not show the numbers red & white
Each no its own colour
Tends to quick recognition
Like a continuation of the material used for counting.
Is smaller. used in larger quantities
Brilliant & fascinating
More extensive work.

How to get them to work with Bead Stair

You might ask: how will you get them to work with the bead bars?, The teacher who asks this should turn ~~the child~~ back & get him to finish his work with the rods.

In very many schools they give the long rods for too short a time – one for understanding & not for exercise.

But if they have not only understood the rods but done exercises with them, digested them. So thoroughly that it has no more to do with them then you will see the child take to the ~~rods~~ beads with pleasure.

Will put many in a row eg.

8 + 3 + 4 + 6 + 4

Will do Additions & Subtractions. Additions which can be abbreviated by multiplication

Above 10 - 10 - 20 etc

The Passage from 10 - 20 has already been begun by the child with the additions on the rods. We can repeat the same with the Bead Bars.

The Tens to be represented by separate Cards when one passes fr 10. to 20 one begins to count the Tens which are represented in separate numbers with different colours.

The Ten Bead Bars can be used as Units

$\boxed{10}$ $\boxed{20}$ $\boxed{30}$ etc.

| || |||

Geometric Form Begins to Appear

The children will see little by little the shape changing as the rectangle becomes wider & wider [Old Friends again!] We can note this visual impression of a rectangle wh widens & becomes a square

Thus beginning to give a foundation for the Geometric Idea of Number.

Composing Numbers w. Cards. etc

If the child wishes to add 4 to 30 we have two cards

$\boxed{3\,|\,0}$ $\boxed{4}$

III ⋮

So the number 34 is not formed of two ciphers placed beside one another but of 30 and 4. – which correspond to exact quantities in the beads

Thus any combination is seen in this manner

It is not two figures put side by side but two well-known quantities 50 and 9.

This gives too an intuition of the idea of zero – nothing – yet gives the position

Similarly $\boxed{100}$ $\boxed{70}$ $\boxed{4}$

Each figure stands for a definite quantity – not an abstract number

The scope of ~~these~~ ~~the~~ exercises is to present the material not an exact succession of exercises.

[At this place. Birds Eye View of Decimal System]

The Long Chain

Recall that in Early Centuries it was difficult to express mathematically large quantities.

By this means can have a clear vision of a large quantity.

1000 Chain gives idea

1) Quantity
2) Relation.

Thus First 100 Chain + 10 Tens w. see the connexion..

The length gives clear idea of correspondence

Thus uniting 10 chains of 100 we get one long chain. The length gives a great impression + on each division ~~no~~ of 100 we can put a card |100| |200| |300| etc

The important thing is to attach attention to the differences in length

Thus.

10

100

1000 This enormously long thing - which goes beyond all customary things - are important impressions to a child

Would a child want to carry it on to 10 times the 1000 chain??

So the 1000 is set as a limit to the actual concrete quantity.

Lines Squares & Cubes

Exposed in this way " looks almost like a Jeweller's Window!" . .

This material does not lend itself to many transformations but to a very clear vision

The particular point is to show

a) The Squares — area.

b) The Cubes solids

(c) The relations between

1) Unit series

2) Squares series

3) Cube series

What is interesting is to see the Relative Differences of units, squares, cubes.

"The Reverse Effect"

It intuitively lends itself to Reasoning — to latent reasoning, which accompanies this observing, testing, proving.

These are for One Aspect of Numbers. Gives the child opportunity to observe different ways of grouping.

Suppose the ~~numbers~~ beads were all loose in a bag. It would not help the mind.

138

Not "Loose in A Bag - Grouped"

This helps the mind - in a way could not do if not grouped. for him.

So Give a Gymnastic

Instead of teaching these things just as numbers: we give a material which contains & presents these concepts;
And we give them to the child at a much more elementary age - so that it can even then come to much clearer concepts than it could even later without the material.

This means that also his progress will be much more rapid.

The Present ~~Knowledge~~ and and in itself

Arithmetic a Liberal Study

We do not give all these ex. on Arithc with the idea of doing sums quicker. (It does help us to do that; but that is not our object.)

Our Aim is to give a Gymnastic to the Intellect to give the Philosophy of Numbers And the more we can realise this the more we can train the mind.

By Educating the Mind we can also — "nebenbei" — arrive at doing sums more quickly.

This way of doing sums was brought by the Arab Traders & certainly is very useful. The Arab merchants used to do their accounts more quickly. And we thank them for it.

But this does not mean we should give the children arithc "in the spirit of a Tapster". They do not do Arithc for the sake of business but for growth. We will give them instead all the easy & slow ways in which they can exercise their intelligence as in a Gymnasium.

['I cannot reckon I have not the spirit of a Tapster!']

We give them these Parallel Exercises — all these elements to clear up the notion of number & form a whole. To make it interesting we divide it into different exercises each of which has its own pleasure

Three Dimensional Knowledge

Example from Arith^c

First comes addition anyhow

9+1   6+3   8+4   3+1   6+1 etc.

Child does them profusely

Afterwards the knowledge becomes organized

Thus the possible groups. – a limitation along a principle – a selection from material already known & spread out – a "drawing up" an emphasising of certain ones. – a clearing up – in short a new dimension.

So with the words. – & the lessons

This is "Red"   This is "green"

This is "long"   " "   "short"

So with the Parallel Exercise on Arith^c. The knowledge is all spread out on the same plain – we must rise above it, master it, organize it, vitalize it, inform it with the immaterial omnipresent intelligence which is totally present in each part. – so that each part becomes related to the whole and change one part & it changes the whole. The mental baggage now becomes a living organism.

148

The Didactic Material

The Principle of the Hook

The same material serves & should serve on first a lower & secondly a higher plane.

Therefore from the first stage the child must have a material so perfect that he can draw from it an experience - yes - but not at first all that is possible.

So the rods are a fundamental material which from the first present to the child those combinations of groups which will be consciously organized later

A Strange Symbolical Drawing

All the Didactic Material should have a little hook - which hooks on to the future mental development of the child.

Many things which the child is to understand later has been placed in his mind by the material already.

- as -

Function Presupposes the Organism

The organ which can function later already exists. ~~These things already exist in the child's mind in a way but do not as yet function - the function will come~~ The function of this material already exists but the organ of the child's intelligence does not yet work - but the material pre-supposes the organism. The things we give the child will function later.

Our Old Friends Again

1149

The Pink Tower age 3 ½

again stereognostic

Again. ~~S~~ "small" "large"

Again Grammar.

adjectives of comparison

Again compared with

Number Cube Tower

Again cubic centimetre

litre volume.

Montessori Edn

A Liberal Education

not

Utilitarian

A Propos. Teaching Arithmetic

Kinds of Interest

a) Utilitarian Those – in social life – who are using numbers in Commerce – cashiers, builders, financiers – always in connection with commerce. (or even applied to other Sciences – but less so)

b) Liberal. Real Mathematics
There the interest is centred on the pure relationship of quantities. It is this (b) higher interest – philosophical – of numbers & their relationships which interests the child

Our Path

In every case then the question is to find the path which leads to the highest interests of the Intelligence

So (in Rods.) we eliminate the useless effort of separating & counting the units; for the child can easily & at once recognise the quantities in the rods

B. (See next page)

Interest & Intellect

Apropos of dry work of counting units into groups –

"Because the child cannot find interest in exercises wh. do not call forth the work of Intelligence

Liberal Edn & "The Problem of Problems"

We must present e.g. Number in such a way that all the mental energy goes straight into ~~re~~ spontaneous research – nothing squandered.

Only so will you see the mind is capable of this liberal interest in truth itself

Then. Not Necessary to give Artificial Problems or little shops with buying & selling.

So **First** We have the interest awakened in number itself – the relations of quantities

and

Second Once this interest is awakened the problem is solved – every problem

Mathe Problems are Facts of Life

Really if this clearness exists with regard to number, ~~prop~~ problems come of their own accord. In life we continually meet w. problems in Arithc. It is difficult to pass a day without one.

Children Invent Problems

It has happened that the children not only discover, but invent Arith problems – began to create them in Compositions because they are facts of life.

If these ideas are carried out we shall see the children writing "composition one day based on emotions, & the next on a problem in Arith^c according to the mood of the day.

Prepare and Await Explosion

So here – as in other spheres – if we awaken the central interest we shall see the explosion of the forces prepared & matured within by discovery in the environment

So. 1). Awaken central Interest
2) Inner Maturity
3) Explosion & discovery in Envt.

Individual Differences

Shall see some developing arith^c minds

"As long as we continue to give problems to the child to solve we shall not be able to solve our own problem

We must therefore 1) give the elements clearly.

that

2) Work may go on & develop itself
3) In such a way as to be of deep interest

This is the Problem of Problems – for if the child is not interested there is no Development

Learning - Two Stages 28/7. 85

Second Stage.

This patient work which is necessary for spontaneous progress

In common method to explain a thing is the main part - here a pt. of departure

Old Method

Yet this explanation is only a superficial detail wh. wd. not lead to any results were it not for

1) pressure of the teacher
2) re-explanations
3) continual showing thro' more or less energetic means.

There must be something more profound & more intimate: it is necessary that the individual the child the "bambino" is to be interested & so the teacher must keep to stimulate it.

But the Interest is in a Material Thing

When the interest comes comes the activity which makes it continue in this open road.

There must be. :- for this work of great patience

a) Interest in the thing
b) Comprehension (no interest without some comp.)
c) Some spontaneous urge to continue in this patient & constant application.

This Patient Work
one has no idea of it. —
slow, simple, continued. —
This is the child's revelation of his natural way of developing by means of ideas taking up also the materials of culture

[cf. Johnny and Greek + Latin, Grammar]

Exercises of Patience

Example from Numbers

This child has already learnt to count fr 1–9. well — has been carefully analysed his work — A number of exercises has already made him master of this counting from 1–10..

So one would think this business is finished — a closed chapter. But it is nothing but an acquisition in which it is necessary a long long time yet.

So when a child repeats again + again a thing he knows well with an obvious interest we have this ~~work~~ virtue of patience — we cannot use another word. Let us then call it Exercises of Patience

87

Going Back to Go Forward

(Johnny again!)

So placing a bead next to [1], .. next to [2]
This would seem - going back. -
Goes back & does simple things again. -
This a new phenomenon - the child who goes back.
(So on ... [3] up to ::: [9])

If we ourselves repeat things we know (just what I am doing with these lectures) we can also make discoveries; and this patient repetition of things we know, which are perhaps already mature in us, (but which still however interest us and hold our attention) this inner urge is something which really exists - otherwise we should not do it. So then something new may come in our spirit, some phenomenon. There are possibilities for us also in this direction, for there is something occult hidden in us waiting to come to light.

We can do a work which brings us beyond the things which are ~~in our knowledge~~ under our eyes (St Thomas) & can lead beyond the things we already know.

Anyway this is thing & you will see it yourselves watching the children.

Mont. Explosion

interesting plan of relationships may spring up: not only knowledge but an intellectual activity.

So — to return to the Material — if we do the same with the thousands will it be a waste of time?

This attitude of the mind which already knows, — which therefore is not forcing itself to ~~learn~~ understand — but wills only to act, Indeed just this slow simple action of putting these ~~objects~~ cards next to these objects is a sort of work — touching, taking putting, arranging in order, contemplating.

There then is the plan which this spontaneous work of the child represents.

[Birds Eye View!]

There is nothing more easy then, ~~than~~ does this work ~~s~~ understanding in its simplicity the constitution of the D System — this grouping together of different units. -

Repeating, repeating, repeating, there are certain particularities of this which come to be understood in a most clear + profound manner. Because one must truly possess these relations in order to become agile in this "gymnasium" like a muscle which repeats certain movements already known — ~~as~~

This is a work which all of a sudden gives us an intuition which makes us go beyond what ~~we~~ ones sees & would be able to suppose. This is the way the child acts, & we are impressed with this phenomenon: that the child just in this spontaneous activity ~~goes~~ is drawn towards an unknown world to discover it.

This is not a theory, a manner of speaking, — <u>it is a fact</u>.

So — To go back to the Example — this patient work is to put in vision this material, which gives the child's ex. view of the Decimal System — putting out the various series of the D.S. — the hierarchies.

. $\boxed{1}$ $\boxed{1111}$ $\boxed{100}$ etc.
.. $\boxed{2}$ $\boxed{1111\ 111}$ $\boxed{200}$
... $\boxed{3}$

You might say but we know this — putting 2 tens next to each $\boxed{20}$ & so on.

Well then having understood does not signify abstention from doing this patient work [we do not leave a person as soon as we learn to appreciate him] — Never mind go on. For the aim of this ex. is not to learn this or that, — but to give an interesting plan of relationships — from which

He who dances well does not cease to dance because he ~~knows how~~ ~~has~~ has learnt well how to dance – he is rather the more interested.

Again when one sings + sings a good note he does not remain mute for the rest of his life: rather feels as if he must continue to sing – because continuing to sing leads to greater perfection of what he has already. + which no one else could give. And this is something which we can attain only when – possessing a thing and an ability which is like a Gift given to us – we can use, develope it by repetition (Like the Talents).

This is what the child's understanding acts upon when it has conquered an interesting thing.

Decimal System. Again

The thing which comes out of this is that when you pass from 9 to 10 you do not add another object, another unity of the same kind, but another kind of object which is a unity in itself.

This is the fascinating thing – because you could add 1 more & have ten separate units, or ten ten bars or ten squares – but in fact you do not.

(contd) ten of these wh. you could bind together - but we don't - these new units are things in themselves. a reality.

These Objects have Names

One can learn the names of these exactly

Difficulties of the Decimal System

Chiefly at the beginning.

a) 1-9. names quantities

b) 10 - 20

After that is easy -

"Whether it is easier to say 2 ~~&~~ dolls or 2 thousand. , 3 pigs or 3 hundreds etc. 5 cherries or 5 tens

Patient Work Again

When these objects are moved patiently, many times, put out in this order from 1-9. - there comes a moment in which the child possesses all the big numbers without any difficulty, & this moment comes suddenly -

The pause comes in counting fr 1-20 - after that everything comes with a rush in an instant.

Composing the Number 1123

1 thousand
1 hundred
2 tens
3. units. –

Now this 1123 is like a sum, ~~just like putting two ten-rods together~~
You see them all distinctly & then put them together – Just like adding 1 table 1 chair 2 stools 3 hassocks. We must no imagine any difficulties where there are none.

The interesting thing is to eliminate all the difficulties & give something so clear that it permits of the realization of this great result which is interesting & represents an an exact idea – in the field of mathematics

This Exercise not Enough – does not explain or teach

The Passage from 9 to 10

We must show again the passage from one group to another – that at 9 there is a jump ~~&~~ a passage from one Category to another – to another unity of a different kind. This too must be matured, analysed by a patient work and thus go to something more & more simple & clear

Parallel Exercises

93

All these contemplations, these studies are of the D.S. ~~+~~ not of arithmetical operations — so it is the nature of the D.S. that we study first.

We can study this D.S. by means of parallel Exercises which are on the same level — but always studying the D.S. — just this ~~e.g.~~ 1–9 (2) this passage to 10.

In an ordinary school they would say the child understands this now & could & should pass on to another matter. But we say no! If it was understood well, this is the moment to remain there, for it is now the work begins.

Here comes the Exercise of Columns

3199
2347
7584

thous	hund	tens	units
....	....	....	~~tens~~ ....

1) Analyse the numbers by means of the card

2) Put in the dots in the columns

In this way we have transcribed in a simple manner this no. What was a noble no. written in figures, placed according to their position in the hierarchy, now has become something inferior — dots in columns. We have gone back in a certain way.

This we have gone back again in a certain manner of speaking.

A Triumph of Ignorance

I wd. say that this ex. is a triumph of ignorance. – going back, a recoiling as we are always doing.

We go forward, we understand, do many exercises – then we go backwards. – just as one goes backward in order to ~~read~~ take a running jump.

This going backwards is of great importance fr. the psychological point of view because then one reposes, & stands as it were continually ~~a~~ over the things one has already learnt & understood & then going backwards one gets up speed for a jump forward.

This is the true way of making progress.

[Adding up the Points in Tens & putting in the next Column]

It is a little work, but a work of attention. People who know how to do the num^e^ operations quickly wd forget a point, one of these passages, – so it is not a question of intellectual acumen, which is in action here but attention – they require a continual vigilance, untiring patience, which never forgets. So with these exercises is formed a mental attitude which is useful in the future as in the present.

Thus etc is an Exercise in the Decimal System not an addition, which a child of 6 or 6½ could do.
It is enough to be able to count up to 10 – as they do in the Casa dei Bambini. nevertheless it is a respectable number which has been dealt with with this exercise of patience, humility, reflection. remaining, brooding over the same thing already learnt – But thus by means of this going back – which is attractive because by means of it the child makes an unheard of advance in doing sums with thousands & thousands.

Simplicity of Dec System

When one understands the D.S. what diffce dealing with millions or thousands.
Enough to know the names of these things which are in the place above – but the names are few to learn. Once this is realized the way is open for progress – without further instruction – The ~~[illegible]~~ garden of enchantment is at the child's disposal for we have given him the key.

Enough to prune away the difficulties to permit the child to make gigantic strides.

Decimal System (Contd)

Addition

Means bringing to-gether - but not in the ordinary way.

Adding quantities according to the same hierarchy.

Adding by quantities with the material.

Subtraction

$$\begin{array}{r} 6349 \\ \underline{4872} \end{array}$$

Note the second figure does not refer to an actual quantity to be made - as in addition.

Also the Stamp-Numbers

Multiplication is Addition & Counting

If we go on counting in another way we count with equal groups

2^3 3^5 4^3 etc

Count them with beads, write, & memorize them.

Writing The number repeated is placed on the margin. Each time we start from the last one - just as if we were counting ones - & write the results on the cards. By repeating many times we end by knowing them by heart -

3 × 9  or 9 × 3

(a) (b)

The result is the same as to the number — but the path of arrival is not the same

In a) start with 3 (brown) & take 9
In b) start with 9 (yellow?) & " 3

The Squares

Doing the numbers beginning with 1, 2, 3, 4 etc. at 2×2, 3×3, ... 6×6 etc we come to a square, visibly. The process is not uniform: at a certain pt we come to the square. For each number it is a special case.—

The Geometric Limit of this Counting by Groups

I could go on to infinity with each number eg 5 · 15 · 20 · 25 .... But we wish to bridle this counting by groups to check it. Instead of going on indefinitely we can go to a certain point. This limit will be given us by the geometric disposition of the numbers.—

Thus we count the 3s till the square is repeated 3 times; fours till the square is repeated 4 times & so on — We set the limits according to limits analogous to those of the Decimal System

There we have always the repetition of the same number & so we are in the field of multiplication

But the same principle we can repeat each time with different limits. It is always a multiples of itself added until it forms a square. – & adding the squares until it forms a cube

So we drive home in different forms the same fact until it is rooted in

Variations by Square & Cube

We can make comparisons between the squares & cubes.

The Pink Tower again in the Cubes

Our old friends again!

Deeper & deeper.

So the quantities are arranged in a Geom^c way. & we can compare.

Linear Comparison

We can now analyse these quantities by observing them in linear form.

We see the quantities increase – squares & cubes – according to a definite law. We can see how the lengths behave.

First The "length of the Squares"! How strange it sounds the "length" of the squares. – & cubes! So we have introduced the idea of arithmetical Length.

In the geometric Comparison we do not realize the differences as we do in the linear form.

June 24th 34.

56

Decimal System Contd

The Web and the Details

We distinguish 1) the Web. – the general structure
2) The smaller details.

If we have our fundamental web. we can do our analytical work on it – so that it can be done independently & separate

The Web or The Scaffolding or Skeleton.

Consists of

a) 1 – 10. Rods etc

b) The same principle extended to the quantities – units, tens, hundreds, thousands

All the exercises in this first part have been absolutely concrete. – the quantities there themselves. in toto.

This structure remains in the mind when we come to counting.

So to repeat

First – the Quantities

Second The Position

The Position

Now it is not the quantity that has to be remembered but the position. –

The Number Frame

This is to show the Hierarchy of Nos.

of Quantities + Symbolic Beads

3654 on Frame and in squares etc

The real difference in these beads apart from ~~the numbers~~ colours is in the position

The Number-Paper

To facilitate the passage from quantities to symbols.

On this paper we do not write 10 — only 1 — but on the next line. Because zero has just this function. And so to make it understood we can do this "moving up to next hierarchy" by the lines — without the zeros.

We always have before us these 9 figures — no more. We must always remember that ten of the units, tens, hundreds cannot be on the same line.

The idea of position becomes clear when we have previously examined these quantities in former exercises.

Which is the more easy? to count 3 tens, 3 hundreds or 3 thousands — In writing 3643. — there is no difficulty but in composing it on the Frame I must remember the reciprocal positions of the numbers.

Addition easy:

$$\begin{array}{r} 3643 \\ 2345 \\ \hline 5987 \end{array}$$

So this material serves to bring the child into relation with large numbers

So this Web. Warp & woof — Two difficulties
1) Quantities 2) Position.

These exercises have to be done separately. The child can count the bead chain & does so now knowing the hierarchy of numbers.

Always only the same 9 & thus a change

127

Number Frame etc

THE PREPARATION

Counting Verbally not Sufficient

To count 1 2 3 4 5 - - - verbally is not enough - not nearly sufficient preparation.

Must Enter the Organism as Second Nature

Comparison with Music

Not enough to say the notes "do" "re" etc. We must repeat it so often in motor activity - on piano that it becomes second nature

This Counting Material. - (up to now) might last for a year or two to the children though the presentation only takes an hour

Example Number Chains

. , 10 bar. 100 chain. 1000 chain.

Whereas to you this presentation might seem easy enough, the child will count over these chains for months and months.

They will have different needs

And will arrange the material in various ways.

After Long Ex. with Rods. Spindles. Discs; Birds Eye view: Bead Stair; Hymn Bk Chart; Squares Cubes etc. -

after all this

Come to the Number Frame

Number Frame

Differs from Ordinary Abacus
Each series a different colour.

Before Presentation

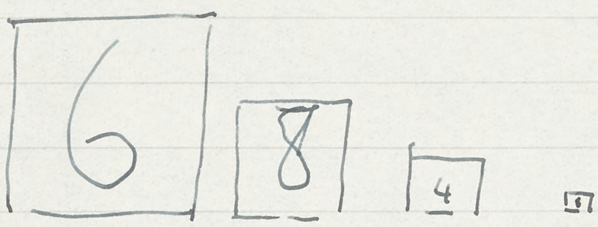
(a) Child must have reached a certain age.

(b) Have done sufficient exercises in counting

Here not Quantity But Position

One might suggest beads of different sizes according to value. But if we reason in this way we should do the same with the written numbers e.g.:-

6 841.



So Our purpose is a material which corresponds to the Numbers in that position alone counts.

The Number ~~Bo~~ Sheets   Zeros

~~Here~~ position is everything. & shown by the lines..

| Zeros are not needed - not necessary |

They would only be necessary if one went on to another kind of paper.. For the zeros show the position.
Only if there is no other way of showing position must ~~be~~ we have zeros

Useful in case of need

The Frame not a Slavery

Our Auth^c boasts of being liberated from the Abacus, fingers etc. & all things material

But this liberation is not necessary in the formative period

Useful because:—

(1). Sees more clearly

(2) Child is able to move (actively)

(3) Works with less mental effort

There is the great advantage in giving the possibility of repetition – the child sees without mental effort.

The child would get tired without the material but not with it. So the child

a) learns more rapidly

b) with less fatigue

c) and is forming ideas at the same time

So the Child Copies the Beads on the Frame on the Sheets

may copy the whole frame 10 or 20 times; Does this in freedom as long as it likes and then

Explosion will come a Sudden leap to large Numbers

Comparison. As the child who has prepared will will suddenly write a whole word, so these children will suddenly compose large numbers up to thousands.

Transition to Millions

Is an easy slip

Will come after a week – always provided there has been no preliminary maturing.

July 8th 1

70

Multiplication

Analysing the Difficulties

are Two main difficulties

1). Memorizing Multiplication Table

2) Whole Series of Difficulties of Reasoning.

All these centre round one main difficulty which is a real understanding of the matter in hand.

These difficulties in No 2. can be analysed & progressively solved. Analysis clears up these difficulties

Let us leave 1) aside as done with –

Further analysis – of Series 2

Difficulty I. ~~Solved~~ Stated algebraically. We must be quite clear as to how we can multiply the sum of two things.

Here are 2 bead bars to be multiplied by 3

9 + 4.. Repeat 3 times

(9 + 4) + (9 + 4) + (9 + 4) I have repeated both these & multiplied all each by 3.

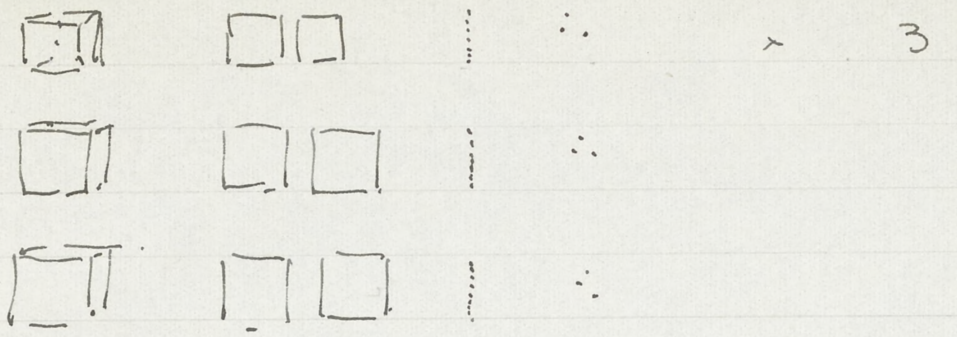
You may say "This is so clear & obvious why dwell on it? Because here is the centre, the k not the crux of multiplication

If I put other groups of beads instead of these it will be just the same.

Thus:–

71

1 2 1 3



3. 6 3 9.

If 1213 is to be multiplied by 3 I must multiply by 3 a) the 1000 b) the 200, c) the ten, d) the three

1000 × 3

200 × 3

10 × 3

3 × 3

Always I do this. So we get first the analyses of the Number to be multiplied

A Step Further

1 4 1 2 × 3. The same thing:–

1000 × 3

400 × 3

10 × 3

2 × 3.

But in this case when I multiply the hundreds by 3 I get 1200 – and this cannot be according to the Law of the Decimal System.

So I must arrange it as

1000 and 200.

10 of the hundreds become a cube and are moved up + 2 squares remain.

So always if we go beyond 9. there is a moving up

72

So we get Multiplication is

a). Multiplying each of these groups wh. have been analysed out.

b) Secondly - Grouping the results according to the Decimal System.

This grouping means that each time we get beyond nine something passes up to a higher part of the hierarchy. -

This is nothing new -

We have repeated this idea more than 9 times!

So we must then - :-

1) Analyse the No

2) Multiply each part

3) Group each result according to the D. System

So the Fact of Memorising remains the same on each level - no more difficult on one than another

The Really Important Thing is to Remember with which group I am dealing. to which part of the hierarchy the numbers I am working with belong

Here comes in the Value of the Number Frame I have only to know the position on this frame and multiplication becomes easy - equally easy for ever part of the hierarchy.

It is always the same thing but in a changed position

The Result is read in succession in all these numbers. Yet I can put them all together & we know by doing this we are doing an addition sum. - For all numbers are numbers & all numbers are additions

Example again:-

2352 × 3.

$$\left.\begin{matrix} 2000 \\ 300 \\ 50 \\ 2 \end{matrix}\right\} \times 3.$$

Its the same as before. 2000 × 3 - the only important thing is to know the line to start on. 2 thousands again:-

3 × 5 = 15. This we know by heart. In which line must I put it. These are 15 tens.
Here the algebraical multiplication gets complicated by the Decimal System because I have to place 15 tens & according to the Decimal System I cannot have more than 9 of one kind.
If I get as far as 10 on a line - hey presto - the whole ten disappear. miraculously embodied in the 1 of the hierarchy above.

These Two Factors Interplay as Warp & Weft of Multip^n

1) The Algebraical Taking of quantities so many times.- addition of quantities so many times
2) The Organizing of the Numbers as they arise according to the Decimal System.

The Dec. System gives rise to this promotion of numbers - but does not change the fundamental basis of multiplication. - always I have to do it in this way.

One group ... so many times
another " .. " "
a third " .. " "

and so on
Nor does it matter where you begin - same thing!

So analysing the Difficulties. – we get

1) Structure of the Decimal System – which has already been overcome

2). The Difficulty of Memory – which we have also done.

3). The Real & Special Difficulties of Multiplication
analysis of groups
keeping to right hierarchy
i.e. the multiplying separately of different groups

This being so – the analysis of those parts which compose our no is in itself the explanation

[So far has been "Short Multiplication"]

The Passage to Long Multiplication

Multiplying by Ten.

$234 × 10. Instead of twice or 6 times
I wish to repeat this no. ten times

~~1000~~ :
200 } 10 = 2 cubes
30 = 3 squares
4 = 4 tens.

By now we know what this means this multipn by 10.
It is a kind of promotion to a higher grade
These three "ladies" mount up & become hundreds
so rising in dignity – these too – all of them

So we have 234 by 10 Without fatigue I promote them all : They all go to a higher grade – like the Birth of a Prince

So they have all been moved up one place.

Multiplying by Twenty

First Move them all up one place.
Second Multiply them by 2.

First I promoted them all 1 degree & then I doubled them.

Thus 234 × 20

$$\left.\begin{array}{r}200\\30\\4\end{array}\right|\ 10 = \left.\begin{array}{r}2000\\300\\40\end{array}\right\}\ 2 = \begin{array}{r}4000\\600\\80\end{array} = 4680.$$

So in Long Multiplication the New Thing is to Analyse the Multiplying Number also

So if I multiply by 20 I must change the hierarchy and double.

If by 300. I change the hierarchy twice and then multiply by 3. or whatever it is.

234 × 200

$$\left.\begin{array}{r}200\\30\\4\end{array}\right\}\ 100\quad \left.\begin{array}{r}20000\\3000\\400\end{array}\right\}\ 2\quad \begin{array}{r}40000\\6000\\800\end{array} = 46800.$$

Thus we shall always have the same multiplication exercise 4 × 6 & so on : but the real Problem is to know what line we are working on.

To take another example

234 × 356. ~~p. t~~

What have I to do? I must break this knot. – these knots – this & that

$$\left.\begin{array}{r}200\\30\\4\end{array}\right|\ \times\ \begin{array}{r}300\\50\\6.\end{array}$$

I shall begin with patience to do all these - because I am simplifying this - (by giving a bit & sending them down!)

Thus :- 200		20000			60000
30	100 × 3	3000	× 3	:	9000
4		400			1200
200		2000			20000
30	× 10 × 5	300	× 5		1500
4		40			200
200		1200			1200
30	× 6	180			180
4		[illegible]			12

By this analysis I overcome or am prepared for the difficulties. The important thing is to pay attention to the line. This revolution is the fault or the merit of the Dec. System. Not the fault of the analysis or multiplication.

The Dec Syst. is master or director of Numbers

a The real difficult is to determine on which plane or hierarchy we are working on. After that it is easy. It is always the same after we know in wh. hierarchy we are working.
b Beyond this the Revolutions caused by the D.S

Thus Multiplication is done with Things - from which you pass on little by little & all these operations are reassumed in a brief & concise way..

Working with the Material all these little Sums disappear & become one thing

Multiplication

Up to now Counting with Separate Groups

Now Counting with Separate Groups wh. are Alike with the Multiplication Boards.

Psychology of Multipn Boards

Something to remain interesting long enough to memorise the results - Aim.

The Interesting Complications

The way of making this interesting has been almost to complicate the necessary means.

also by

Giving an imposing appearance -

eg. a portfolio something to be tied up - a packet - something mysterious containing interesting things with promise of the future.

Large quantity of multipn Forms in diff. colours but small enough to limit the exercises

Thus :-

a) A small clear dose

b) A simple movement - in pulling beads on the cards.

Also the No in the little Window

(Really unnecessary after the first 3 ; but we are adding a complication. + Discs

It does not help in the multiplication

but serves to give a Succession of movements which hold the attention long enough for him to learn & memorize.

Our main purpose is that the child shall repeat the ex. many times spontaneously So there must be interest

All these Little Facts, which seem unimportant to us, form a group of movements which hold the childs interest in a work itself uninteresting

Thus when the exercise 1) is very simple mentally + 2) a combined series of movements.
It becomes restful (cf my present work!!)

Can go on with this exercise with little mental effort & quiet movements for a long time

We are as it were binding his mind around this very simple work & the highest thing he can find to do is to memorize it.

Experience shows they will go on for a long time - more than three two sheets.
Not obliged to work on successive days but the material remains exposed for choice.

Memorization of Tables

What is interesting is that this calm & monotonous ex. seems to lead spontaneously to memorization.

We find the children spontaneously studying the cards to memorize them.

Sometimes they form little groups & ask each other.

Sometimes they go to the teacher & tell her

3 × 3 = 9.!

Correction of Error

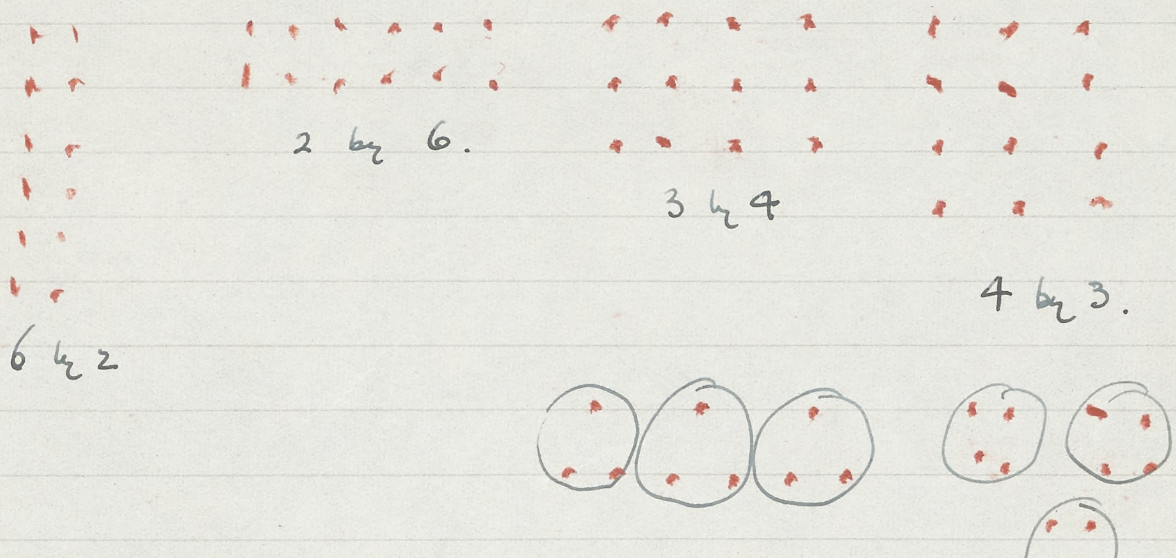
We give a card with all the Tables so that they can compare & correct their own

A note of exactness & patience.

Playing Ball with Arithmetic

We give certain exercises — wh. are not simply learning by heart but developing elasticity of mind. A sort of game.

(1) Thus To lay and arrange a given no. in various ways:—



And so on.

(2) Division a) Take a chance no. of beads ~~fr~~ in a bag & count them —
2) Then arrange in rows of threes.
Eg 20 goes 6 times & 2 over

The Opposite Finality to Multipl^{n}

1) In Mult^{on} we do not know what the final number will be
2) In Div^{n} we know the big no. but do not know how many groups there will be.

So Another Portfolio. with Division Cards

The Embryonic Stage

Thus (see Multipn. Board) we have the elements of multipn + Division all united in one exercise.

The Explanation is given at a later stage. ie that multipn + Division are the same movement reversed but here the child finds it out intuitively

The question of the Remainder – after a difficulty – is here seen visually

This material represents all these things – Multipn Division, Factors, Divisibility of Nos Prime Numbers etc – in an embryonic stage they are in a simpler form.

Also – as in an embryonic stage – it has many different things under the same appearance Similarly in Reading. Writing + Drawing

Then In Developement

– these things divide + specialize. + draw apart to their respective functions

Here we have in an apparent unity all these things potentially different, wh. will afterwards go off on their own paths. which is the way of Evolution + Development.

Thus we see Multiplication as progressive sums – group by group. Division is identical, but the opposite similarly

Also Divisibility of Numbers

also here - embryonically - is something else - the Divisibility of Nos - the search for factors which comes a long way later.

No one would think of teaching this now - as such - to the small child - but what is there in this game of bulle if not to show the different factors.

Thus this fine distinction

3 by 4 or 4 by 3 is made clear here by the position

Then to our arrangement of groups in 12

2 by 6, 3 by 4, 4 × 3. - means we can ~~go~~ divide into these factors.

So the beginning of the Divisibility of Numbers.

When we try to find what groups make 12 we are finding the factors; but we do not talk of these things - but -

We have put the child on a path where they are evident & simple

In this First Period then we do not only find the first learning of Multiplication but we see also the beginning of what will develope later

This Brings Greater Interest into the Present

For it is like a sign post or a road leading on over the mountains, leading to a new & interesting country !

What comes later - this road over the mountains - is the gradual complication, little by little, of these things - the determinating + separating of the one from the other.
- as in other fields.

Reading separates from Writing
And Drawing becomes - Art.

This is the Course of Every Development
First a Simple Embryo where all functions are united together - not in confusion - but more vague and indistinct
Then comes specialization + successive complications

of The Inferior Forms of Life
The Protozoa.
has all the functions of life
respiration, reproduction, digestion
Sensibility - all in one cell

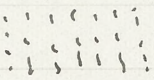
Division

To Divide Equal Parts among Individuals

The important thing is that each child should have his part. Done by a little group.
If anything is over to be discarded.

Example

I take a certain quantity of beads 77. to divide amongst 4 people. If I am faithful to the Decimal System I cannot take more than 9 of each kind.

So 77 ÷ 4 people. 

1 ten 9 units . 1 over.

The Quotient (quotiens)

When we say 77 ÷ 4 = 19 this is wrong. ~~It equals~~ because the original number remains - it is merely divided up. The quotient is the share of each of the individuals - True one is left over but the original no remains. ~~E~~ Simile If you divide your hair into two parts the hair still remains

Again 684 ÷ 4 = 1 hund. 7 tens 1 unit
The Original no remains & we can get it by putting all together again.

So division & proving division are done by taking away & putting back..

Thus by Repetition Two ideas become clear
1). The Quantity is merely subdivided
2) Each of the units receives the same quantity

Division by 12 (& also 122)

12 People –
a) 10 of them choose a representative
b) Two single units (simplici!).

And so Representative with ribbon same colour as the tens. receives 10 times the amount received by the units. At end the Rep. divides his into ten parts.
So the important thing is what the unit gets.

Collective Work

This is a collective piece of work. – requires representatives & money-changers. Done in this way it may seem a complicated business but is not so complicated as it seems. It requires the attention of a number of people & organization. – Repeated a number of times it makes clear:–
a) The Quantity is merely subdivided – not destroyed or diminished
b) Each unit has the same quantity.
There is rigorous division. just distribution. & each has the same share. The Representative has more – but only as representative

But what about Individual Work?

But it would become too complicated if we went on in this way? What would happen to the Individual Work?

Division – Individually

For the Indiv. work we later take these cards.
Thus 27 ÷ 4.
Put 4 in the slot & arrange the beads in "pairs"
6 rows & 3 over.

~~The~~ Transition to Division Boards

We could do it as above with a large no of beads – but would be very long & tedious. So instead of taking 10s 100s 1000s in quantities which would be very heavy to use we use symbolic beads of different colours. e.g. green for unit, red tens, blue 100s

Comparison

Put units yellow   next green units
tens "   " red tens
squares 100 "   " blue hundreds
Thousands –   " green units etc.
Thus compares the quantities with symbols

We could Divide 123 From Quantity to Symbol
a) ~~Quant~~ Multiplication Board (Two pairs)
b) With Quantities –
Representatives Hundred. Tens. & Units
(c) On the Three Division Cards.
Thus the transition.

Thus idea Units. Tens, Hundreds. –
Units (of Thousands) Tens (Of Thousands) Hundreds (Of Thousands)
Can be made clear. Infra

[Geometric Disposition of Numbers

. Dot   Line   Square   (units)

Dot (Thousands)   Line of Tens of Thousands   Square of Hund of Thousands

etc - always in this same configuration.
Pt. line. square - Pt. line square
Always the same progression on higher & higher spheres.

cf The Famous Diagram -]

~~Mat~~ Quantity & Symbols again

If we have recourse to symbols it means a simplification. But if the idea is not clear in thus having recourse to symbols it means obscurity.

Abstraction — So we must keep to the actual facts with regard to quantity until the idea is clear

Because in this way instead of symbols being a complication they will be a help & we shall advance by leaps & bounds.

But if the beginning is not clear every step after is confusing. ~~The~~

[cf In a new region. we need something sure under our feet]

In ordinary schools the First Step ~~a gloss~~ is a (lesser) obscurity & the next of greater. Whereas if we start with light the path leads on to the path of simplification. Then we arrive at the last stage running without anyone to help us without fatigue

By practising in this succession of Things.
first with Things
Then with Symbols.
The children will naturally pass on to Numbers as a simplification:
For Numbers are much simpler than all the rest.
If Numbers are so much simpler why not start with them?.
Because it seems easier to start with numbers but it is really obscure.
But if the children begin with objects and pass to numbers then they find numbers easy.
One who has understood well can progress unlimitedly.
These are all Different Levels on wh. the mind Works

Different Levels
This important to realise the mind works at the same problem on Different Levels.

115

Working on Different Levels

Different minds & the same mind at different ages of its development work on different levels.

Examples

Cubes. – Sensorial
Grammatical
Arithmetical.

Arith^c

1) Sensorial. – Quantity unmeasured except by senses
2) Number – counting
3) Operations.
   a) With quantity only
   b) Quantity & Symbol combined
   c) Symbol alone written.
   d) In the head!

Problems insoluble on One Level Solved Easily on another.

Reasons

a) The mind is above the other ? (or itself forming)
b) It has a wider range of experience
c) It has organized knowledge
d) It sees the parts in relation to each other & the whole – & sees the Problem as one of these parts
e) The Fanatic or specialist. – the one-sided man – The Scientist. etc.

p.t.

1) The Doctor sees yr. malady in relation to the whole of yr. body - its right functioning.

2) The mechanic of your car - knows its parts in relation to the whole -

3) The financier -

4). The Priest - Christian. - The whole includes the next life -

It is experience vivified, organized, related ~~Sta~~ illuminated - made one interrelated whole by the informing intelligence

Collective work.

Silence Game.

Division by 4 - 12 etc

Arithmetic

Long Thous and Bead Chain.

Squares + Linear Cubes etc

Geometry

Sensorial insets with cards

Reading Commands

Naturally Hunt in Cupboles

Algebra

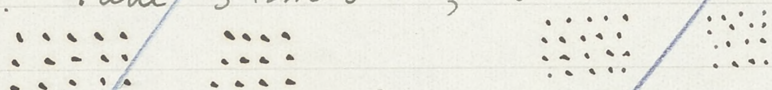
Introduction (See Same world & Different Levels)
Its the same apparatus; but with a mind more mature.

Recall the Inner Structure of Multip^n

$$(5+4)\times 3 = \left.\begin{matrix}5\\5\\5\end{matrix}\right\} + \left\{\begin{matrix}4\\4\\4\end{matrix}\right.$$

Or $123 \times 3 = (100+20+3)$ taken 3 times

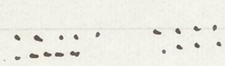
Present Two Bead Bars 5 and 4
Child has already done some of these
e.g. Take 3 times, or 4 times - or 5 times - or 6 or 7.



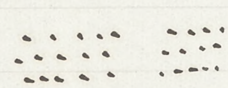
This is then the principle of multip^n in all cases where we have groups to be multiplied
But we cannot stop at 5 or 50 or 500. So we can say
$2(a+b)$

Present Two Bead Bars 5 + 4

$2(5+4)$ =



$3(5+4)$ =



$4(5+4)$ =

This is the principle of multip^n in all cases where we have groups to be multiplied.

But we cannot stop at 4 or 5 or $6(5+4)$
So we can generalize & say. - ~~letting a any number~~.

$$2(a+b) = 2a+2b$$

This is the general fundamental statement.
The mind wh. has understood this principle has made a mental generalization

Second Presentation $(a+b)(c+d)$

$\quad\quad a \quad b$

I take $(5+4)$ & repeat it $(3+2)$ times

=

So instead of saying $(a+b)$ taken 5 times we say $(a+b)$ taken $3+2$ times

$= 3a + 3b$

$\quad 2a + 2b$

The thing we have to learn is the manner of proceeding. We take the first group first 3 times & then 2 times.

What is the Use of Brackets? To hold together things which should be held together

So $9 \times 5 = (5+4)(3+2)$

or $(5+4)$ taken c times

$(5+4)$ " d "

or $(a+b)$ taken c times : $ac + bc$

$(a+b)$ " d times $ad + bd$.

So $(a+b)(c+d) = ac + cb + ad + bd$.

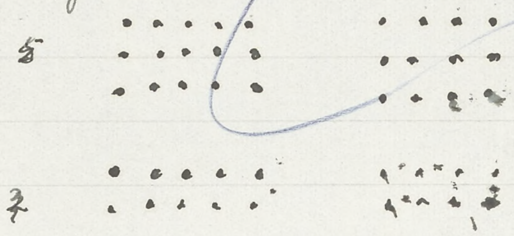
So in order to enter the Study of Algebra there are few things to be changed. We put letters to indicate general quantities.. Because we are not on the look-out for numbers - but on the look out for what happens to the facts - the innermost facts

We are interested not in the end of the journey but in the road that leads to it

Second Presentation $(a+b)(3+2)$

Take ~~$(a+b)$~~ $(5+4)$ & repeat it $3+2$ times

I get :- 5 4



3 times

2 times

We have no longer a precise interest in the number itself – no longer these big precise numbers – but now it is the inner mechanism which interests us.

Thus take 5×9. It is not the 45 which interests us in all this reasoning. What interests us is to study the movement of these inner molecules. It is the study of the function which we are making.

If we for a moment study the letters of the alphabet with brackets enclosed as parts of a quantity.

If we can get over this first difficulty we shall find ourselves in a field which is very clear & limpid. There is nothing different now ~~in our~~ outwardly – in our looking at these little groups & discs & the letters of the alphabet are not different.

Where then is the difference? It is in the maturity of the intelligence which is looking on.. The things to show & to do are simple. The only thing is to reach the level on which they can be understood.

If once this idea is taught there is nothing else to teach. The understanding of it is a matter for the student. Have we not always thought of the letters of the alphabet as more simple & clear than numbers. Then we have taken a step back?! 7×9 is difficult. But this is more easy. We have come into an easier field: it is the Intelligence that Triumphs here – that is all. If you have eyes you can see.

Let us see if you have understood

$5(a+b+c) = 5a+5b+5c \quad (a+b)(c+d) = ac+bc+ad+bd.$

Squares or The Second Degree

Let us see what happens when we multiply a number by itself.

eg 5×5 — Even if we have no memory we know because

5×5 = 5 squared

$7 \times 7 = 7^2$ (squared) Even if we don't know we are just as clear. If the no. is multiplied by itself there is always a way.

& we can get out of it & cut a brilliant figure! If we are asked in an examination what is 8×8 we can say it is 8^2 squared & we are on a higher level. We shall be even more intelligent if we can put a sign instead of the number, thus:—

$$a \times a = a^2$$

Now Perpend:— I am very ignorant & cannot count to four: so I shall wriggle out of the difficulty. $5 \times 5 = ?$

$5 \times 5 = (3+2)(3+2)$

$= 3 \times 3 + 2 \times 3 + 3 \times 2 + 2 \times 2$ — which is

$3^2 + 2(2 \times 3) + 2^2$

But I will slip further out — how clever! — & say

$(a+b)(a+b) = (a+b)^2$ (shorter to write)

$= a \times a + ab + ab + b \times b$

$= a^2 + 2ab + b^2$

Now I will give this a Resounding Name — the Square of a Binominal — $(a+b)^2$.

Squares Cont.

To Go Forward We must Go Back

Those who go on although first by going back. Really it is going deeper into a thing superficially learned. Eg the mind of St Thomas Aquinas always going back to the Bambino Gesù whom he learnt first at his Mother's knee.

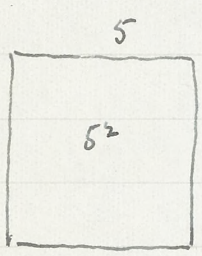
Again. Newton & the apple. Watt & the Kettle.

So we go back to the simplest figure — The Square.

We know the Relation of Numbers to ~~Geog~~ Geometry.

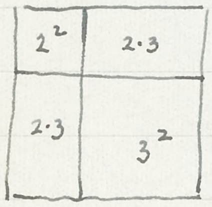
5^2 = square of 5.

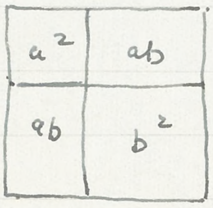
Have also learnt area of surface — length by breadth



But $5 = 3 + 2$: so we can have $(3+2)^2$

So





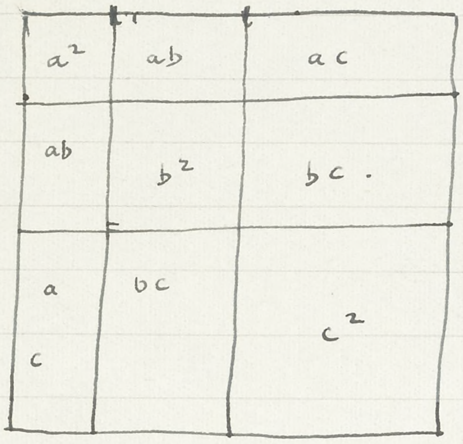
$(a+b)^2$

This is a visual fact clear on the surface

This is easier than Arith^c. & the combination of letters is seen at a glance. more easy than ever with letters instead of numbers

Now we can do the square of a trinomial And the mechanism of the operation is exactly the same. Requires a bit more patience — but we have been acquiring this for a long time

$$(a+b+c)(a+b+c) = (a^2+ab+ac)+(ab+b^2+bc)+(ac+bc+c^2)$$
$$= a^2+b^2+c^2+2ab+2ac+2bc.$$



Arithmetic Progression

Now we come back to the Number Rods

First counting material

We remember there is some special in this series. $9+1 = 10$

$8+2 = 10$

$7+3 = 10$ etc.

But we might go on to any number & do the same eg $19+1 = 20$

$18+2 = 20.$ etc

Let us count the 10's we have made in rods

$5 \times 10 + 5$ over.

So $1+2+3+4 \cdots\cdots 10 = (10\times5)+5$

- Now in Homage to the Decimal system I want it all in tens !:-

So $\frac{10^2}{2} + \frac{10}{2} = \frac{10^2+10}{2}$

What I don't like is this 10 - it so limits me. The same would happen if it were 1000

So instead of 10 I put any no. – If I don't know

~~any one~~ a persons name. I.e. – "anybody".

So $\frac{n^2+n}{2}$ –

Summarising these.

$$1+2+3+4 \ldots\ldots\ldots n = \frac{n^2+n}{2}$$

So now we have the general formula. & can work out any no. without labour.

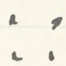
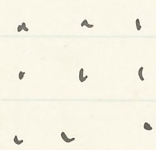
So we come little by little to philosophise about numbers & quantities & are able to generalise And thus we become "detached from all material"

Square Root

Square Rt

Mult° always can be shown under the form of a rectangle — & the square is only a particular case

Here I have square cont.g 100 beads —
If consider side of sq. 10 units.
= Sq root

Now I set myself a problem.
If I have a certain no of units & I want to find √ — I
place the pegs to form a square —
1) smallest possible square.
ie of 2 
2) Then go on
putting them in such a way as to form a square
— next of 3
3) 

The next of 4 & so on. —
When I have finished placing the units
I find the sq rt of the 20 is 5

You see tho' I have done 3 diff't operations the result has been the same.
25 in form of a square

The Sum of all the units composing these cubes is equal to the natural series of nos squared

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2$$

The quantities can be moved in 2 ways
Either uniting a) in bigger groups
b) or splitting into smaller

and in equal or unequal groups.

Similarly if split can be split in equal or unequal groups

Group Addition
Each child can bring a quantity + a card
Subtraction
Each a card only

In Multiplication They brought equal amounts

16 III/C

3

Algebra Arith° + Geom

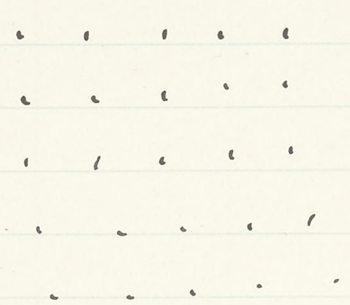
We continue the study of algebra & arith° because Algebra helps us to understand the mechanism of the diff° operations.

These subjects are linked together by the fact that they both express themselves in geometric form.

Relat^nsp. Between these Diff^t Operations

Squaring app. with holes.

I multiply 5 × 5



The no 5 repeated 5 times gives a square

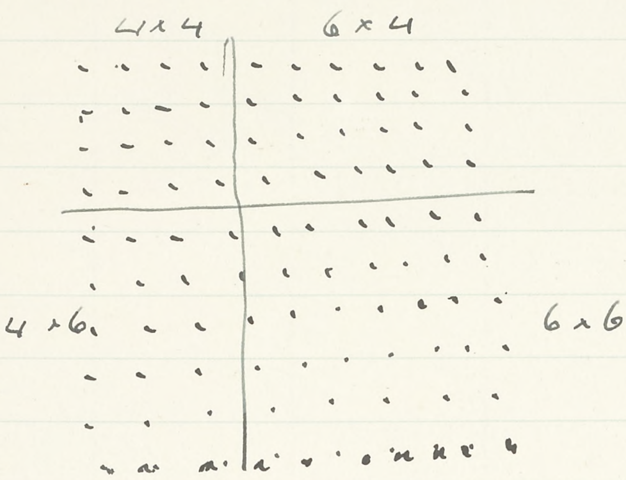
Division. I wish to divide 25 by 5.

D.M. makes a row of pegs, then another row underneath until the pegs are used up. Result same as before

This fact shows how closely related are the facts of × & ÷.

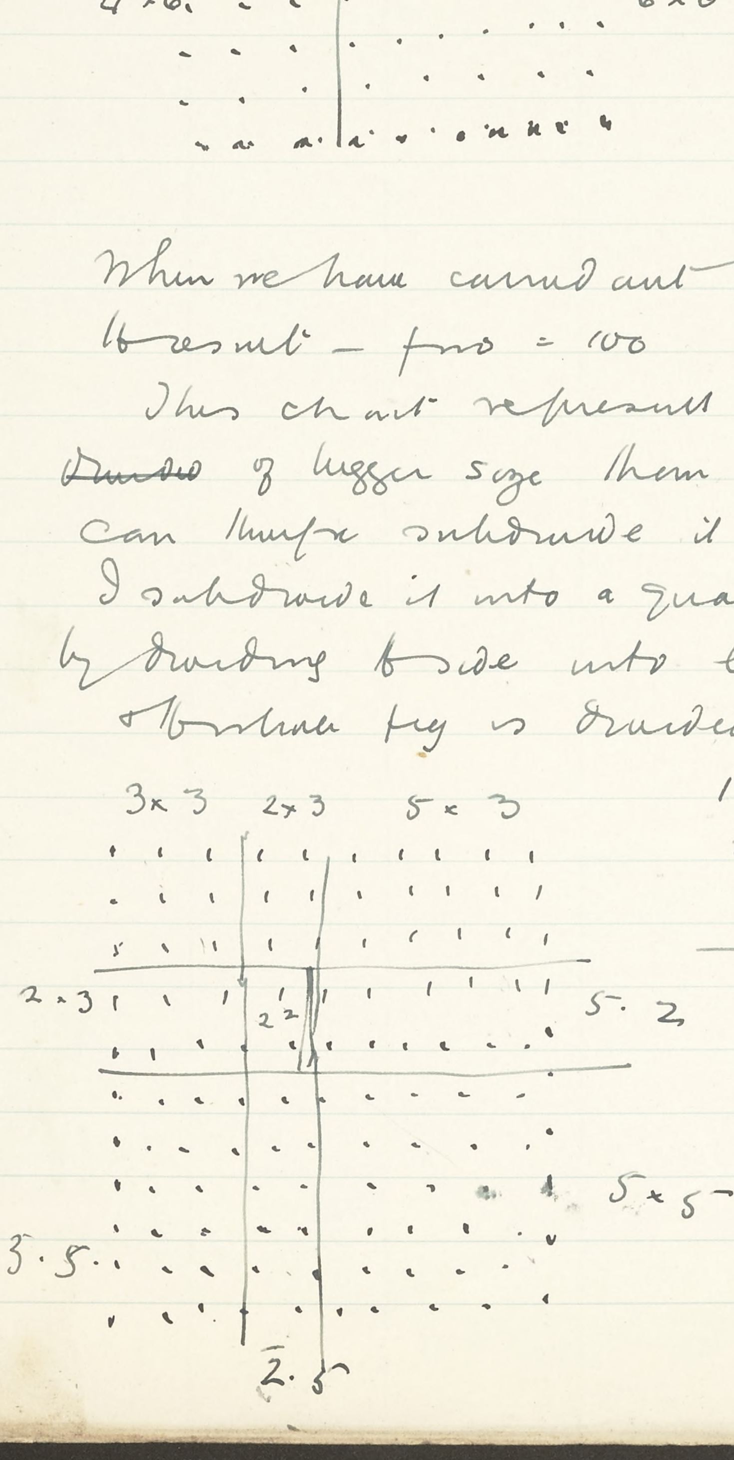
Also shows why to prove a ×. we do division & vice-versa

Now to carry out an analysis of this square of 100 beads. – i.e. divide it into smaller parts by means of strips of paper. & carry out the smaller operations which are indicated
~~The value of~~ by the divisions I make
The value of this square, as you know, is ten times ten; ~~Tb~~ & if I subdivide the side of the square into 2 parts I have multipns



4 × 4 = 16
6 × 6 = 36
6 × 4 = 24
4 × 6 = 24
100

When we have carried out the multn & added up the result – find = 100
This chart represents the 100 square. It is ~~Divided~~ of bigger size than the bead square & I can therefore subdivide it into more parts
I subdivide it into a quantity of small figures by dividing the side into lengths 3 2 & 5
& the whole fig is divided into parallels



1st a square . 3²
2d . 2²
5 2
} all diagonals

Rectangles

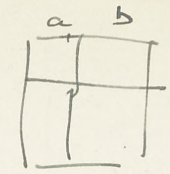
1) The squares on diagonal
2) Several rectangles – 2×3
have 2 alike correspᵍ to sides
of first square

3) Then 5 × 3 = 3 × 5
also 2 × 5 5 × 2
= 100

Exᶜᵉ Children divide the squares into as many
ways as possible – interesting thing is they
always come to the same no
[at first we gave head squares with thread
to divide them: but did so many times
we made printed forms]
Then larger ones
15 units a side

In diag. these 2x. something became evident
wh. is independent of the numerical quantity +
reveals a general law
1. If any 2 divisions – 1 shorter than other
always same pattern
a) 2 squares diagonally
b) 2 rectangles flanking them
~~No matter what the divis~~
No matter what the division of the sides of
the square may be 2 + 7 or 3000 + 510 (?)
always same geometrical pattern
represents a Law
(geom. to the help of arithᶜ see beginning)

To express algebraically $a + b$



$a + b = a^2 + b^2 + 2ab.$

The Geom^l representⁿ of this formula gives rise to a great many applications & gives opportunity for work to be carried out w. coloured paper & other materials so that the construction of the geom fig remains fixed in c. mind.

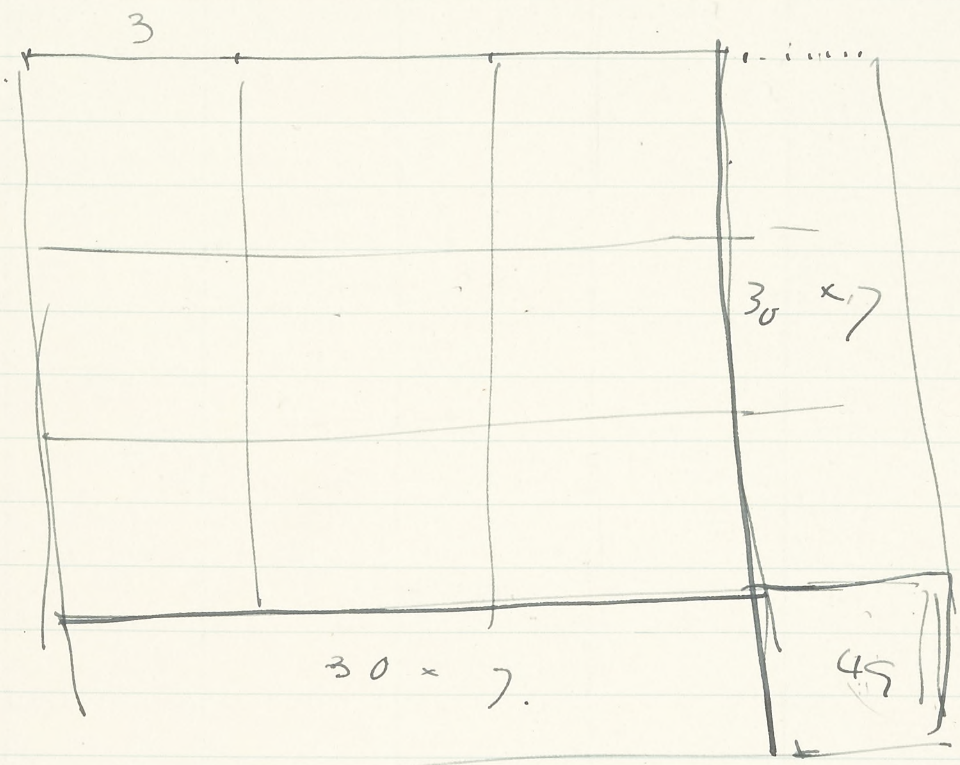
10 + 7

17   17 ~~819~~

<u>Decimal System appⁿ</u>

Letters stand for quantities a tens b units.

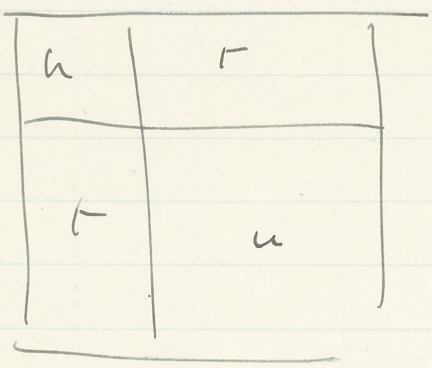
This square is divided into 2 parts & represents 3 tens & 7 units.—
In first square we have $30 \times 30 = 900$
This space occupied by 10 ~~tens~~ 10 & that is by hundreds



= 1369

What remains fixed by these new steps into Arith^c is the relative values that occupy each fig
a   top sq = 100 hundreds
b   2 rectangles tens
c   Bottom sq units

Now a number is a sum of units arranged in groups of decimal category & if I distribute them as shown by this square. that is hund on top. tens sides. units bottom. — I sho be able to construct a sq. whose sides have the value of the sq. root of this no



Let us see if we can carry it out with mat.

529. —

We take 5 squares of 100

2 10

9 loose beads

I distribute these in the order given. I must first arrange in the form of a square the hundreds. —

I can form a square with 4 — 1 over.

for I sh need 5 more for next square — only 1 —

So —

Break into 10 bars —

gives me 10 + 2 = 12

Place these bars 10 on both sides of the square

& we can use them them up exactly.

Now only sq. (u) to fill in —

So no. is by reading the last line

23

So have found $\sqrt{}$ without any calculations just by placing out quantities according to plan

So Next Stage with Symbols

instead of actual quantities. —

So with the Peg Board 1024

(1) I take red colour to rep. hundreds, blue tens, green u.
I make a square with 10 hundreds using 9 red pegs + 1 over. —

~~(2)~~ I change 1 into ~~ten~~ tens — ten blue pegs i.e. .. 2 along them

32

Now can do arithc Problems

1) First thing to find sq root of the first group. i.e. how big a square can be built with ten hundreds
Biggest to be found is 3.

2) Place the pegs represt tens on the inner sides of sq. as long as we have any.
(To do this by calculn is more diffic.

If we were to state the rule for abstracting the sq root we shd say we must ~~first~~ square the first symbol found & subtract it from the no of hundreds. Thus we do more simply with apparatus.

Now we have to distribute the tens over the 2 sides of the square. The no. at our disposal is 12 with which we must make rectangles. but of these rect. we only know one side — ie of the sq. We must therefore place pegs rep 9 tens on the inner sides of sq. as long as we can.

[Rule. Divide the quantity of tens I have at my disposal by 2 times the part of the root already ~~found~~ found — same numerical value as the side of the square already built.

W. Abstract quantities is hard

with material only necessary to put the quantities in their respective places & he learns —

(3) Symbolic Pegs —
more difficult but easy enough

(4) Only nos & words — difft & bilingual.

Thus algebra geometry & arithc — united in this first period help to understanding of relations.

Devocations

If arrested devel. arrested power of mot.
Idiot can't speak
Importance of SP for Incarnation

These energies revealed either
1) Chief wanting to act
& 2) St attract^n for Emot.

Show manifest in
Activity. guided by will.

Energetic C

1) psychic energies developed independent of mot (fantasies)
2) mot detached to psych energy. (disorder)

Submissive

Obedience in acts of introduction

Slowing up & loss of Certain Const we instincts

2y order.
loses attraction to Emt.
no observation.